Is there a Daily Discount Rate?
Evidence from the Food Stamp Nutrition Cycle

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Abstract

Quasi-hyperbolic discounting predicts impatience over short-run tradeoffs. I present a direct non-laboratory test of this implication using data on the nutritional intake of food stamp recipients. Caloric intake declines by 10 to 15 percent over the food stamp month, implying a significant preference for immediate consumption. These findings constitute a rejection of the permanent income hypothesis and are extremely difficult to reconcile with exponential discounting. The data support an explanation based on time preference and reject several alternative explanations, including highly elastic intertemporal substitution. I explore implications for the optimal timing of transfer payments under alternative assumptions about preferences.

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Consider a consumer who is indifferent between enjoying one additional dollar of consumption today and 99.6 additional cents of consumption tomorrow. Such an individual has a daily discount factor of 0.996, and if she is an exponential discounter her annual discount *factor* will be about 0.23 (corresponding to an annual discount *rate* of about 146 percent). She would therefore strictly prefer 24 dollars of additional consumption today to 100 dollars of additional consumption one year from now, and would happily accept seven cents today in exchange for 100 dollars in five years.

As these calculations illustrate, an exponential discounter who is reasonably patient in the long-run must be almost perfectly patient in the short-run. Even small amounts of daily discounting translate into enormous amounts of annual discounting in the exponential model. By contrast, the quasi-hyperbolic model of discounting (Laibson, 1997) severs the link between short- and long-run time preference, and predicts significant present-bias even in short-run trade-offs.

In this paper, I test for the presence of short-run impatience using data on the caloric intake of food stamp recipients. I find that the average caloric intake of members of recipient households declines by 10 to 15 percent over the food stamp month. A calibration exercise shows that, to be resolved with exponential discounting, these facts require an annual discount factor of 0.23 (or an elasticity of intertemporal substitution of 6.7). Survey evidence on household financial circumstances reveals rising desperation over the course of the food stamp month, which suggests that a high elasticity of intertemporal substitution is not a likely explanation. Additionally, households with more short-run impatience (as estimated from hypothetical intertemporal choices) are more likely to run out of food sometime during the month, consistent with an explanation based on time preference.

The data can reject a number of alternative hypotheses. Households that shop for food more frequently do not display a smaller decline in intake over the month, casting doubt on explanations based on the depreciation of the household's food stock. Data on household composition and inter-household transfers of food indicate that the observed decline is not the result of strategic interactions within or between households. Finally, the evidence indicates that the monthly nutrition cycle does not result from households' confusion about the value of their food stamps.

This paper makes several contributions. First, my findings constitute direct field evidence for short-run impatience. While quasi-hyperbolic discounting has been applied to a wide range of economic issues (see, e.g., Angeletos et al, 2001; Cutler, Glaeser and Shapiro, 2003; Gruber and Koszegi, 2001; and O'Donohue and Rabin, 2001), evidence on short-run discounting has derived mainly from the laboratory (Frederick, Loewenstein and O'Donohue, 2002).[1] Having reliable, real-world values for short-run time preference parameters is essential to conducting simulations of savings policy experiments (Laibson, Repetto and Tobacman, 1998).

Second, in showing that the timing of consumption is sensitive to the timing of payments, these results constitute a rejection of the Permanent Income Hypothesis (PIH). The rejection is especially striking given that households must solve the same problem each month for a series of months. Although other research has argued that the PIH is violated at monthly frequencies (Stephens, 2002 and 2003), such work has tended to use data on expenditures, rather than actual consumption, to test the PIH. The difference between food purchases and food intake may be especially important at high frequencies, where the durability of purchases is more likely to be a concern.[2] As the PIH has important implications for the evaluation of fiscal policy (Poterba, 1988), it is important to study whether existing rejections result from data deficiencies of this sort.

[1] Exceptions include DellaVigna and Paserman (2001), DellaVigna and Malmendier (2002), and Laibson, Repetto and Tobacman (2003), which infer hyperbolic preferences from job search behavior, health club plan choice, and life cycle consumption and savings facts, respectively.

[2] Goods such as restaurant meals for which expenditure and consumption occur almost simultaneously raise the concern that food eaten in the home is a very close substitute for food eaten out of the home. My data address *total* food intake, thus avoiding many of these issues.

Finally, the food stamp program is of interest in its own right (Currie, 2003). The economic literature on food stamps has investigated labor supply effects (Fraker and Moffitt, 1988) and the effects of cash-out (Moffitt, 1989; Whitmore, 2002), and has put relatively little emphasis on the timing of benefit use.[3] My findings indicate that the timing of payments may have important consequences for the welfare of recipients. To the extent that my results tell us something about underlying time preference parameters, these policy implications may generalize to other government transfer programs.

The rest of the paper is organized as follows. Section 1 describes the data on caloric intake, as well as other supporting datasets used in the paper. Section 2 presents the basic evidence on the behavior of caloric intake over the month. Section 3 calibrates an exponential model of intertemporal choice and discusses evidence on the elasticity of intertemporal substitution. Section 4 presents direct evidence on the relationship between time preference and food intake, and shows results rejecting a number of alternative hypotheses. Section 5 calibrates a quasi-hyperbolic model of the allocation of the food budget, and discusses the consistency of the quasi-hyperbolic model with the observed facts. Section 6 discusses policy implications. Section 7 concludes.

1 Data

In a typical month in 2001, some 17 million Americans received food stamps, with the average household getting roughly \$160 per month in benefits. This average household had \$620 in gross income and contained 2.3 individuals.[4] I will make use of three datasets in my analysis, each appropriate for studying different aspects of

[3] See Wilde and Ranney (1997, 2000) for notable recent exceptions.

[4] For a review of the characteristics of food stamp participants and participant households, see U.S. Department of Agriculture (2002).

the behavior of food stamp recipients.

The best available data on the consumption patterns of food stamp recipients comes from the Continuing Survey of Food Intake by Individuals, 1989-91 (CSFII), conducted by the U.S. Department of Agriculture (USDA). The CSFII is based on a nationally representative sample of households, and collects three days of dietary intake data for each household member. The first day of data is collected by 24-hour recall: the respondent is asked to list everything she ate on the previous day, with interviewer prompts designed to help respondents fill in frequently omitted items. The second two days of data come from intake diaries completed by respondents over the next 48 hours.[5] USDA staff process the data to produce aggregate measures of nutrition, such as total daily caloric intake. Along with fairly rich demographic and health data, the CSFII contains information on whether respondents are receiving food stamps and, if so, the exact date of the last benefit payment. From this variable I construct a measure of the number of days since receipt of food stamps.[6]

For some purposes it will be convenient to know the market value as well as the nutritional characteristics of food eaten. The Nationwide Food Consumption Survey 1987-88 (NFCS) contains detailed information on the quantity, price, and characteristics of food used in the home over a seven-day period for a representative sample of households. Participating households were contacted at the beginning of the diary period and asked to keep a record of all foods used in the home over the next seven days. This information was then collected by an interviewer at the end of the seven-day period. For food eaten out of the home, total expenditure is recorded.[7]

[5] Enns, Goldman and Cook (1997) offer further discussion of survey methodology and a descriptive analysis of time trends in food consumption.

[6] Because the question about the date of last benefit payment is asked only on the first survey day, it is possible that a household at the beginning of its benefit month could be coded as being at the very end of its month. Since virtually no states vary the benefit date systematically by month, I correct for this problem by assuming that the day of the month of the next benefit payment will be the same as the day of the month of the previous benefit payment.

[7] See Huang and Lin (2000) for a further description of the dataset.

I will also make some use of the Evaluation of the EBT Expansion in Maryland, a survey conducted in 1992 and 1993 to document the effects of Maryland's statewide adoption of the Electronic Benefit Transfer (EBT) system.[8] The survey was conducted in two waves, the first prior to the implementation of EBT, the second after its implementation. It covers a random sample of $1,298$ food stamp households residing in Maryland. In addition to basic demographics, respondents were asked a variety of questions about their use of food stamps, including a rich battery of questions on the costs associated with using food stamps. Data from the second wave has been matched with administrative records of all transactions debited to the benefit card during an associated month. I will be using data from the pre-implementation wave of the survey, as this corresponds more nearly with the time period of the CSFII.

The Appendix Table presents summary statistics for variables of interest from these three datasets.

2 Monthly Patterns in Food Intake

Table 1 presents regressions of the log of total caloric intake on the number of days since the household's last receipt of food stamps using data from the CSFII. Since this dataset contains at most 3 days of intake records per person, there is insufficient power to estimate this relationship with individual fixed effects. I therefore adopt the identifying assumption that the difference between the interview date and the date of last food stamp benefit receipt is distributed randomly across households in the sample. This allows me to recover the relationship between caloric intake and time since receipt of food stamps using cross-sectional variation.

In all specifications I use dummy variables to control for the year, month, day of the

[8] See Wilde and Andrews (2000) for a further description of the data and an analysis of the use of different sources of income over the month.

week, day of the calendar month, and day of the survey. All reported standard errors have been adjusted for possible within-household correlation in the error structure. In these and all subsequent regressions, I use the survey weights recommended by the data providers to adjust for nonresponse.

Column (1) of Table 1 reports results with no demographic controls. Caloric intake declines by a statistically significant 0.45 percent per day after receipt of stamps. Column (2) includes dummies for gender, race, ten-year age categories, and the log of height in inches (with a dummy for missing data on height). Adding these controls increases the fit of the model considerably but has only a tiny effect on the coefficient of interest, moving it to a 0.40 percent daily decline.

One potential problem with these estimates is that survey measures of energy intake are subject to severe underreporting (Livingstone and Black, 2003). If underreporting is additive–if each respondent reports true intake less a constant–specifications using the log of caloric intake as the dependent variable will yield inconsistent estimates of underlying parameters.

Column (3) of Table 1 therefore repeats the specification reported in column (1) using the level of caloric intake in place of the log of intake. The estimated daily decline is strongly statistically significant. When divided by the mean of caloric intake of roughly 2500 kilocalories per day for the U.S. population in 1989 (USDA, 2000), the estimate gives a daily decline of 0.32 percent.[9] This result is quite similar to the coefficients reported in columns (1) and (2), suggesting that imperfect measurement of caloric intake is not likely to be introducing significant bias into my estimates.

Since the quality of food can vary quite independently of its quantity, it may be inappropriate to use energy content as a summary measure of food intake. As I show in section 3 of the Appendix, if food is a composite commodity produced

[9] Section 2 of the Appendix discusses formally the issues surrounding the functional form of underreporting, and shows that in the case of additive measurement error these estimates of the percentage daily decline will be consistent for the true slope parameter.

from several inputs, then under sufficient regularity conditions the total market value of food intake will at an optimum be proportional to the amount of the composite commodity consumed.

The NFCS reports a household-level measure of the market value of food consumed over a seven-day period. Column (4) presents results from a regression of the log of this quantity on the number of days since receipt of food stamps as of the first day of the sample period.[10] This regression includes dummies for Census division, household size, interview month, interview year, and day of the calendar month (at the beginning of the sample period). Results are weighted as recommended by the data providers.

As column (4) shows, the decline in caloric intake is indeed indicative of a decline in the overall value of food consumed. The percentage daily decline in food value of 0.73 is larger than that observed for caloric intake, reflecting substitution towards lower-cost foods over the month. The fact that two datasets with completely independent samples–the NFCS and CSFII–both reveal a statistically significant decline in food consumption over the month demonstrates the statistical reliability of the monthly nutrition cycle.

3 Calibration of the Exponential Model

Towards a calibration of an exponential discounting model, consider a consumer who maximizes utility over calories C given by

$$U = \sum_{t=0}^{T} \delta^t u(C_t) \tag{1}$$

[10] Well-known survey fatigue effects make it likely that the total value of food consumed reflects food consumed at the beginning of the sample period more closely than food consumed at the end. Using the number of days since receipt of food stamps averaged over the entire seven-day period reduces statistical precision but gives qualitatively similar results.

where $u(\bullet)$ is increasing and everywhere strictly concave and t indexes days in a month of length T.[11] Suppose that the consumer cannot borrow against future income and is therefore given a fixed stock W to spend over the month. Within the month, the consumer faces a gross interest rate R, possibly less than unity, and a deterministic stream of prices P_t, possibly constant. The budget constraint is therefore given by

$$W = \sum_{t=0}^{T} \frac{P_t C_t}{R^t}. \tag{2}$$

This model yields a standard Euler equation:

$$u'(C_t) = \frac{P_t}{P_{t+1}} \delta R u'(C_{t+1}). \tag{3}$$

In the special case of isoelastic utility in which $u'(C_t) = C_t^{-\rho}$ for all t, equation (3) can be written as

$$\Delta c_{t+1} = \frac{r-\gamma}{\rho} - \frac{\Delta p_{t+1}}{\rho}. \tag{4}$$

Here, lowercase letters denote logs, Δ denotes changes, and $\gamma = -\log \delta$. From the evidence in section 2, we know that Δc_{t+1} is roughly -0.0040 for the average household. In order to translate this into an estimate of the daily discount rate γ, we must first pin down values for r and ρ (assuming within-month variation in prices is zero on average). While food stamps and food in the pantry do not accumulate interest, it is possible that food depreciation could create a motive for early eating. I investigate and reject this possibility in section 4.2, and therefore assume that $r = 0$.

It remains, then, to establish whether there exist reasonable values of both γ and ρ that can fit the facts. As equation (4) shows, ρ is equal to the inverse of the intertemporal elasticity of demand for calories, sometimes called the elasticity of intertemporal substitution. A one percent increase in the price of calories tomorrow relative to today will lead to an increase in relative caloric intake of $\frac{1}{\rho}$ percent.

[11] Note here that I am assuming perfect certainty. As Section 4 of the Appendix shows, this is a conservative assumption, in that it will tend to bias any calibration toward a discount rate that is too low.

Suppose an annual discount factor of 0.8, which is well below recent estimates based on the life cycle profile of consumption, even for low-income groups.[12] This corresponds to a daily discount rate γ of about 0.0006. In order for this to be consistent with my finding of a 0.40 percent daily decline in caloric intake, ρ would have to be on the order of about 0.15, corresponding to an elasticity of intertemporal substitution of about 6.7. In other words, if the price of a calorie becomes 10 percent lower on Tuesday relative to Monday, the caloric intake on Tuesday of a typical food stamp recipient would have to increase by 67 percent relative to Monday. This would seem to imply an enormous ability to substitute calories across days.

If we instead assume $\rho = 1$, which corresponds to log utility, the implied elasticities seem far more sensible: a 10 percent decrease in relative prices will correspond to a 10 percent increase in relative intake. With this calibration, however, the daily discount rate implied by my estimate of the drop in consumption over the month is about 0.0040. This corresponds to an annual discount factor of about 0.23, far below what would seem to be the reasonable range of values. (A person with a discount factor of 0.23 would accept 130 dollars of additional consumption today in exchange for 100 dollars of additional consumption every year forever.)

These simple calculations suggest that, given the observed decline in consumption over the month, food stamp recipients must be either implausibly impatient in the long run or extremely willing to substitute caloric intake across days. In the next subsection I show direct evidence that high elasticities of intertemporal substitution are not responsible for the monthly patterns in food intake.

[12]Laibson, Repetto and Tobacman (2003) calibrate the discount rate to the mid-life wealth-income ratio and find a discount factor of .91 for the least patient group (high school dropouts). Gourinchas and Parker (2002) use the method of simulated moments to fit a dynamic model of consumption to the life-cycle profile and find discount factors consistently above .93 for several occupation/education categories.

3.1 Highly Elastic Intertemporal Substitution

Inferring time preference from a declining consumption path relies crucially on the assumption of diminishing marginal utility from food. If calories today are very good substitutes for calories tomorrow, then even a tiny amount of time preference could lead to rapidly diminishing food intake over the month.[13]

Similarly, if households' preferences are non-convex over certain regions, then standard models of intertemporal choice may lead to false conclusions about preference parameters. In particular, suppose that households like to have a few feast days each month and a number of days with relatively less intake, rather than having the same amount of intake on all days. In this case, even slightly impatient households might well have all of their feasts at the beginning of the month, since the cost of feasting is presumably constant over the month. This type of model could therefore produce a steep decline in caloric intake over the month without unusually present-biased preferences.

Models of this kind can produce declining *total* utility over the month. However, even if there are regions over which marginal utility is non-diminishing, an optimizing agent will generally consume in a region of diminishing marginal utility (Becker, 1971). Moreover, if such an agent is patient in the short run, her marginal utility will not rise over the month–otherwise there would be an opportunity to increase total utility at the margin by reallocating consumption towards later days.

The central prediction of these models is therefore that the *marginal* utility of food (or the marginal utility of money to buy food) does not rise significantly over the month, even though the amount of food eaten declines. Evidence of rising marginal

[13] To illustrate, consider the extreme case of linear utility, which corresponds to infinitely elastic intertemporal substitution (or $\rho = 0$). With no impatience, the agent will be indifferent between consuming 2000 calories today and 2000 calories tomorrow, or 4000 calories today and zero calories tomorrow. However, any arbitrarily small amount of impatience will lead the agent to strictly prefer 4000 calories today and zero calories tomorrow over any other allocation with the same total number of calories.

utility over the month would reject the models discussed above and provide additional support for a model based on time preference.

Respondents to the Maryland EBT survey were asked the following question:

> Suppose you had a choice between getting $50 in cash one month from today, or getting *less* than $50 today. Would you take less than $50 to get the money today?

The 20 percent of respondents who answered affirmatively were then asked:

> What is the smallest amount of cash you would take today rather than getting the $50 one month from today?

The mean imputed monthly "discount factor" for the sample of respondents is 0.93. While the usual caveats about hypothetical choice data apply (Wallis and Friedman, 1942), the answer to this question will likely capture both time preference and the extent of the household's need for cash. Respondents who are impatient, or who find themselves in difficult financial circumstances, are likely to reply that they would accept less than $50 today in exchange for $50 in a month. Since the time preference component is presumably stable over the month, any relationship between the preferences expressed in response to this question and the number of days since receipt of food stamps would strongly suggest changes in the marginal utility of money over the month.

As Table 2 illustrates, the data reveal a striking monthly cycle in households' willingness to accept a smaller immediate payment in exchange for a larger future payment. Column (1) shows the results of a probit regression in which the dependent variable is a dummy for whether the household would accept less than 50 dollars today in exchange for 50 dollars in four weeks. The estimates indicate that the share of households willing to make this trade increases by 0.35 percentage points per day. Column (2) adds additional demographic controls, including dummies for age categories, gender, race, and whether the respondent is a high school graduate, leaving the results essentially unchanged. Columns (3) and (4) repeat the specifications of columns (1) and (2), using as a dependent variable the log of the "discount rate" implied by the respondent's reply to the hypothetical choice question. The results indicate that this "discount rate" increases by 0.24 percent per day over the course of the food stamp month.

These findings show that food stamp households find themselves in increasingly dire straits as the food stamp month progresses. This fact contrasts sharply with a model in which a calorie today is just as good as a calorie tomorrow. It is also difficult to reconcile with explanations based on rotting food or other forms of depreciation. It is, however, quite consistent with an explanation based on time preference: impatient consumers will allow their marginal utility of consumption to increase over the month, because they are unwilling to sacrifice eating today in order to prevent a shortage tomorrow.

4 Robustness

Both the magnitude of the decline in caloric intake and the pattern of rising household need for cash over the month make fitting an exponential model to the data extremely difficult within the reasonable range of parameter values. In this section I will argue using both direct evidence on time preference and indirect evidence on alternative models that the food stamp nutrition cycle is due to short-run impatience.

4.1 Direct Evidence on the Role of Time Preference

The most direct test of the role of time preference would be to estimate the effect of cross-sectional differences in the discount rate on the magnitude of the monthly decline in caloric intake. Unfortunately, the CSFII and NFCS do not contain any direct measures of time preference. I therefore take an alternative approach, which is to use evidence from the Maryland EBT survey's hypothetical choice between receiving 50 dollars in four weeks or less than 50 dollars today. This survey question, discussed in greater detail in section 3.1, provides a useful estimate (or at least a correlate) of the true rate of time preference.

The Maryland EBT survey data do not contain information on caloric intake. However, the survey does contain some questions related to the variability of spending and food availability over the month, making it possible to ask whether more patient individuals smooth more effectively. For example, respondents were asked

> In [the previous calendar month], did anyone in your household skip any meals because there wasn't enough food, money, or food stamps to buy food?

Nine percent of respondents answered "yes" to this question. Column (1) of Table 3 shows the results of a probit regression of the answer to this "food insufficiency" question on a dummy variable for whether the respondent would accept less than \$50 today in exchange for \$50 in one month. Individuals who would accept less than \$50 today are seven percent more likely to report having run out of food in the previous month. As column (2) shows, this finding is robust to the inclusion of a set of demographic controls, including dummies for age categories, gender, race, and whether the respondent is a high school graduate.[14]

Though suggestive of an effect of time preference on households' ability to smooth food consumption, these regressions raise concerns of reverse causality. Households that are having a bad month–in the sense that they have run out of food–may be more

[14] Very similar results can be obtained using a number of alternative dependent variables, including the number of days in the previous month on which the respondent worried about having enough food.

willing to accept less than \$50 today in exchange for \$50 in one month. Although the food insufficiency question asks about the previous month, households that experienced food insufficiency last month are presumably more likely to be experiencing it now. Food insufficiency might therefore be causing high estimated impatience, rather than impatience causing food insufficiency.

To address this issue, columns (3) and (4) re-estimate the specifications of columns (1) and (2) controlling for the amount of food stamps remaining out of the household's most recent allotment. The coefficients on the impatience dummy do fall slightly, but they remain statistically and economically significant.

Overall, then, the evidence indicates that time preference plays an important role in households' smoothing of food consumption over the month. However, there are alternative models that could potentially explain the monthly pattern in nutritional intake. In the next subsections I evaluate several of these models and find that none is consistent with the available data.

4.2 Shopping and Depreciation

Possibly the simplest explanation for the consumption decline is that food rots. Suppose that a household were to do most of its major grocery shopping on the day on which it receives food stamp benefits. If stored food loses some percent of its value for each day it goes uneaten, then even a perfectly patient household would display a consumption decline over the month.

The CSFII data contains a survey question on shopping frequency. Respondents were asked how often someone does a major shopping for the household, and were able to answer either more than once a week, once a week, once every two weeks, once a month or less, or never. Table 4 repeats the baseline specification of Table 1 on four shopping frequency categories.[15] If anything, the evidence suggests that households with frequent shopping have more of a decline in consumption than households with infrequent shopping.

The variation in consumption patterns across shopping frequencies could certainly be due to other differences between households. Nevertheless, the results in Table 4 show that even households shopping on average once or more each week display a significant decline in caloric intake over the month. Evidently, the food stamp nutrition cycle is not caused by infrequent shopping and food depreciation.

4.3 Strategic Motives

If individuals can extract resources from one another, either through transfers or through outright theft, then food stamp participants may effectively face negative interest rates. The more wealth I consume today, the less will be available tomorrow to be stolen or transferred to others, and the more I can credibly demand from those who care about me in order to prevent my own starvation.[16]

The most straightforward effect in this class of models is direct theft. If each dollar of wealth carries with it a per-period probability of theft, this hazard rate will tend to make a declining consumption profile optimal. Respondents to the Maryland EBT survey were asked whether any food stamps were lost or stolen in the previous two months, and if so for the unrecovered amount.[17] Using information on the amount of food stamp benefits received, I have converted this amount into a daily rate of loss. The median daily amount of loss and theft is 0, and the average is 0.0001. This figure is far too small to contribute significantly to the observed consumption decline.

[15] Households that report never doing a major shopping trip are omitted from the analysis.

[16] This is similar to Becker's (1998) observation that we may sometimes prefer to cross the street in order to avoid feeling obligated to donate to a beggar.

[17] Since some of food stamp recipients' wealth is held as food rather than food stamps, it would be desirable to know the rate of theft or loss of food. Unfortunately, this information is not available, although the probability of theft of food would have to be quite considerable to alter my conclusions.

A scramble for resources within the household could potentially generate a downward-sloping consumption path. In such a model, when benefits arrive, each household member tries to eat what she can to capture as much of the available resources as possible. One simple test of the importance of this theory is to compare single-person households to households with multiple individuals. Only 8 percent of the observations in the CSFII are from individuals in single-person households, making precise comparisons difficult, but it is still possible to ask whether the results are driven by large households.

Table 5 addresses this issue. Column (1) presents the baseline specification of Table 1, interacting the number of days since receipt of food stamps with the number of individuals in the household (top-coded at 6). While individuals in larger households do appear to experience a larger decline in consumption, the difference is statistically insignificant and quantitatively quite small. Adding an additional household member increases the rate of consumption decline by .02 percent. And as column (2) shows, for respondents in single-person households the daily decline in caloric intake is quantitatively similar to that estimated for the average respondent in Table 1. Though the estimate is quite imprecise, it does not indicate an important role for within-household competition in determining the time path of consumption.

An alternative approach to examining consumption behavior directly is to ask whether, in the Maryland EBT survey, individuals living in larger households tend to report skipping meals more often than respondents in multi-person households. Columns (3) and (4) of Table 5 address this question in a probit framework. Column (3) presents a univariate analysis of the relationship between household size and self-reported food insufficiency. There is no appreciable effect, and if anything the estimates suggest that individuals living in smaller households are more likely to have run out of food in the past month. Column (4) adds demographic controls to the model; the coefficient on household size is now positive but extremely small.[18]

Even if within-household competition is not an important factor, transfers of resources *between* households may, play a role. If I know that the less money I save the less I will be expected to contribute to others, and the more I will be able to credibly demand from friends, my consumption may decline over the month even if I am perfectly patient.

Measuring the magnitude of this effect is difficult even with perfect data, because in a completely symmetric world no transfers will actually take place. Fortunately there is considerable heterogeneity in reality: not all members of the relevant community will necessarily be on food stamps, and in many states those who are receive their benefits at very different dates.

In particular, the transfers model predicts that food stamp recipients will be net donors of food (or other resources) earlier in their food stamp month (when their wealth exceeds the community average) and net recipients later on. These implications are testable, since the CSFII records whether a respondent's meal is eaten at another person's home.

Table 6 explores the relationship between time of month and eating in the home of friends or relatives. As column (1) shows, there is no correlation between the probability of eating at another person's home and the number of days since receipt of food stamps. This is not simply a matter of lack of statistical precision: the probit model can reject positive effects as small as 0.07 percent per day. In column (2), I examine the relationship between the probability of eating in another person's home and whether the respondent indicates being short on cash on the survey date. There is no evidence that food eaten out of the home buffers shocks experienced by the household. Column (3) uses the number of calories obtained in others' homes

[18] Results are similar using a dummy variable for single-person households rather than a continuous measure of household size.

as the dependent variable. Although the amount of food transferred increases over the month, the effect is tiny and statistically insignificant. Column (4) repeats this specification using a dummy for whether the respondent was short on cash as the independent variable and suggests the same conclusion.

On the whole, then, the data reject explanations for the decline based on strategic considerations. While these concerns may be present, they are not quantitatively strong enough to show up in the data, and therefore seem unlikely to be the driving force behind the food stamp nutrition cycle.

4.4 Confusion about the Value of Food Stamps

Liebman and Zeckhauser (2003) have recently suggested that recipients are confused about the value of their food stamps. A household that must supplement its food stamps with cash may think early in the month that food stamps are worth less than their face value equivalent in cash, leading the household to overconsume food early on in the month. Later, when food is being purchased with cash, the household will correctly value the food and will consume less. As a test of this hypothesis, which Liebman and Zeckhauser have colorfully dubbed "Schmeduling," I have compared households for whom food stamps cover all household food expenditures to households that spend more on food than they receive in food stamps. The results indicate a small and statistically insignificant reduction in the monthly nutrition cycle for households that use only food stamps for food (regression not shown). It does not seem that confusion can account for the monthly decline in caloric intake.

5 Calibration of the Quasi-hyperbolic Model

Having shown that the weight of the evidence strongly supports a time preference-based explanation of my findings, it remains to compare the exponential and quasi-hyperbolic discounting models in terms of their ability to replicate the observed consumption patterns. In section 3 I argued that it is not possible to construct a sensible calibration of a standard exponential discounting model to explain my quantitative results. In this section I present and calibrate a quasi-hyperbolic model of intertemporal choice that is consistent both with survey evidence on the time preferences of food stamp recipients and with the existing literature on non-exponential discounting.

Quasi-hyperbolic discounting posits that an individual at time t discounts utility from a future period $t+j$ by the factor $\beta\delta^j$. In other words, tomorrow's utility is $\beta\delta$ less important than today's utility, and utility two days from now is δ less important than utility tomorrow. With a high δ and a low β, an agent will be simultaneously patient over long-term choices and impatient over short-term ones. With $\beta = 1$ hyperbolic discounting reduces to standard exponential discounting.

To fix ideas, consider our T-period consumption problem in which we abstract away from uncertainty, interest, and variation in prices. As before, let utility be isoelastic with parameter ρ. Then the hyperbolic Euler equation (Harris and Laibson, forthcoming) at time t is given by:

$$C_t^{-\rho} = \left[C'\left(W_{t+1}\right)\beta\delta + \left(1 - C'\left(W_{t+1}\right)\right)\delta\right] C_{t+1}^{-\rho} \tag{5}$$

where $C(\bullet)$ is the consumption function. Since the model incorporates conflicts between the present self and the future self, the lower is the tomorrow self's marginal propensity to consume out of wealth, the more this condition will approach the standard exponential Euler equation.

In the isoelastic case, consumption is proportional to wealth, so we can write

$$C\left(W_t\right) = \alpha_t W_t \tag{6}$$

where $\alpha_T = 1$ and

$$\alpha_t = \frac{\alpha_{t+1}}{\alpha_{t+1} + \left(\delta\left(1 - (1-\beta)\,\alpha_{t+1}\right)\right)^{\frac{1}{\rho}}} \tag{7}$$

for $t < T$. This provides a simple recursive method for solving the equilibrium consumption path.[19]

It is now possible to ask whether sensible values of quasi-hyperbolic preference parameters can fit the observed decline in caloric intake. For parsimony, I will assume throughout that $\delta = 1$, which is likely to be a very good approximation at daily frequencies. Laibson, Repetto, and Tobacman (2003) estimate the annual time preference parameters of a hyperbolic discounter solving a life cycle consumption problem and find $\beta = .7$ for their benchmark model. In order for this estimate to be consistent with the decline in caloric intake, we will need $\rho = 3.4$. This implies that a 10 percent reduction in relative prices causes a 2.9 percent increase in relative consumption, which does not seem unreasonable.

If we assume log utility, then the quasi-hyperbolic model can fit the consumption decline with a β of 0.91. This is within the range suggested by experimental evidence (Frederick, Loewenstein, and O'Donoghue, 2002), and is at the high end of the range of estimates reported by Laibson, Repetto, and Tobacman (2003). It is also very similar to the average monthly discount factor of 0.93 imputed from the hypothetical choice evidence discussed in section 3.

It is thus possible to replicate the key features of the food stamp nutrition cycle using a parametrization consistent with life cycle facts, survey evidence and experimental results on quasi-hyperbolic discounting.

[19] See section 5 of the Appendix for a proof that this is the solution.

6 Policy Implications

In the quasi-hyperbolic model, the equilibrium consumption path is suboptimal in the sense that the consumer would be willing to pay for a commitment technology that would force her not to over-consume in early periods. This begs the question of whether there might be a policy tool for reducing the monthly consumption decline. Since the introduction of EBT, which drastically reduced the cost of delivering benefits, some have suggested increasing the frequency of payments (Wilde and Ranney, 2000).

This is not an unrealistic proposal: In New York State, Temporary Assistance to Needy Families (TANF) benefits are already paid out twice each month. And in a focus group conducted in 1990 some participants suggested breaking benefits down into smaller amounts delivered more frequently (Ohls, Fraker, Martini and Ponza, 1992):

> "Break it [benefit check] down with a check on the 1st and a check on the 15th like they used to do [for AFDC]. If you're spending most of your money in the first few days of the month with one check, if you spend most of it on the 1st, you know [with two checks] you got that other money coming through so you're stringing it out."

> "Give it to us in two installments. At the end of the month I'm dying [for money]. If you got it on the 1st and 15th, or whatever, it would be so much better. Checks or coupons, it doesn't matter, either way, but it does not last a month. The second part of the month is always a struggle."

It is important to note, however, that the magnitude of the realized welfare loss depends crucially on assumptions about the elasticity of intertemporal substitution. For the case of log utility ($\rho = 1$), a consumer whose "true" optimal consumption path is constant over the month but whose actual consumption declines by 0.4 percent per day suffers losses equivalent to 0.06 percent of her total monthly food expenditures, or about 15 cents per month for the typical food stamp household.[20] For $\rho = 3.4$, the case consistent with Laibson, Repetto, and Tobacman's (2003) preferred estimate of β, the losses are only 0.2 percent of monthly expenditures, or about 50 cents per household per month. On the other hand, if the marginal utility from food is very rapidly diminishing, say with $\rho = 40$, then the losses are more substantial: 2.1 percent of monthly food outlays or 5.25 dollars per month.

These potential gains must be evaluated against the increased administrative and other costs associated with more frequent benefit distribution. While obtaining a precise estimate of these costs is beyond the scope of this paper, some rough calculations are possible. In Maryland in 1993, after the introduction of EBT, total food stamp program costs per case month were about $3.85 in contemporaneous dollars (Logan et al, 1994). If an increase in payment frequency were to raise costs by, say, five percent, we would need a ρ of approximately 1.3 to justify the policy change, assuming that an increase in payment frequency could completely eliminate the consumption cycle.

These calculations suggest the need for further evidence on the elasticity of intertemporal substitution. If consumers are relatively happy to substitute calories across time periods, the welfare losses may not be large enough to justify an intervention. This would be consistent with Cochrane's (1989) observation that the welfare effects of deviations from the optimal consumption path tend to be small in practice. Alternatively, if consumers are unwilling to substitute food consumption across days, the potential for gains is large.

[20] These calculations assume that a typical food stamp household spends $250 monthly on food, roughly the median of reported spending in the CSFII. See Appendix 6 for the formal model underlying the welfare calculations.

7 Conclusions

The evidence presented in this paper challenges the PIH, and exponential discounting more generally, as descriptions of the intertemporal choices of food stamp households. In contrast, the quasi-hyperbolic model is consistent with my findings, and has potentially very different implications for the expected impact of fiscal policy and the optimal timing of benefit payments. Whether my results generalize to other populations is an important topic for future research. Data on nutritional intake of the sort I have employed here are likely to be an important tool for future studies, because they allow direct observation of consumption at the daily level. Such a high level of detail is especially important for testing theories, such as quasi-hyperbolic discounting, that focus on trade-offs made over the very short run.

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Table 1: Monthly Patterns in Food Intake

Dependent variable	(1) log(caloric intake)	(2)	(3) caloric intake	(4) log(food value)
Days since receipt of food stamps	-0.0045	-0.0040	-7.9439	-0.0073
	(0.0021)	(0.0019)	(3.1772)	(0.0038)
Demographics?	No	Yes	No	Yes
Dataset	CSFII	CSFII	CSFII	NFCS
Number of observations	6652	6652	6652	309
R^2	0.0796	0.1895	0.0966	0.6013

Notes:

In regressions using the CSFII, each observation describes one household member on one survey day (out of three total days). In regressions using the NFCS, each observation describes one household on one week. Regressions (1) to (3) include dummies for survey day, day of week, year, month, and calendar date. Regression (2) includes dummies for gender, race, ten-year age categories, and the log of height in inches (with a dummy for missing data on height). Regression (4) includes dummies for Census division, household size, interview month, interview year, and day of the calendar month (at the beginning of the sample period). In regressions (1) to (3), standard errors in parentheses are adjusted for intercorrelation within households. Survey weights used as recommended by data providers.

Table 2: Monthly Patterns in the Need for Cash

Dependent variable	(1) Pr(would accept less than \$50)	(2)	(3) log(willingness to accept)	(4)
Days since receipt of food stamps	0.0035 (0.0014)	0.0033 (0.0014)	-0.0024 (0.0011)	-0.0024 (0.0011)
Demographics?	No	Yes	No	Yes
Model	Probit	Probit	OLS	OLS
Number of observations	1100	1100	1100	1100
(Pseudo-)R^2	0.0058	0.0178	0.0040	0.0132

Notes:

Data from Maryland EBT study. Each observation describes one respondent. Probit coefficients reported as marginal effects. Demographic controls include dummies for ten-year age categories, sex, race, and high school graduation status of household head. Dummies for missing data are included for all controls.

Table 3: Time Preference and Food Insufficiency

Dependent variable: Pr(skipped a meal last month)

	(1)	(2)	(3)	(4)
Would accept less than \$50 today in exchange for \$50 in four weeks	0.0655 (0.0249)	0.0578 (0.0236)	0.0616 (0.0244)	0.0522 (0.0230)
Amount of food stamps currently remaining (\$100)			-0.0371 (0.0155)	-0.0389 (0.0149)
Demographics?	No	Yes	No	Yes
Number of observations	1106	1106	1100	1100
Pseudo-R^2	0.0128	0.0346	0.0224	0.0456

Notes:

Data from Maryland EBT study. Each observation describes one respondent. Coefficients are marginal effects from a probit regression. Demographic controls include dummies for ten-year age categories, sex, race, and high school graduation status of household head. Dummies for missing data are included for all controls.

Table 4: Shopping Frequency and Caloric Intake

Dependent variable: log(caloric intake)

Shopping frequency	(1) More than once a week	(2) Once a week	(3) Once every two weeks	(4) Once a month or less
Days since receipt of food stamps	-0.0104	-0.0087	0.0021	-0.0029
	(0.0035)	(0.0033)	(0.0028)	(0.0031)
Number of observations	655	1394	1895	2687
R^2	0.2496	0.2374	0.1135	0.1130

Notes:

Data from CSFII. Each observation describes one household member on one survey day (out of three total days). Regressions include dummies for survey day, day of week, year, month, and calendar date. Standard errors in parentheses are adjusted for intercorrelation within households. Survey weights used as recommended by data providers.

Table 5: The Effect of Household Size

Dependent variable	(1) log(caloric intake)	(2)	(3) Pr(skipped a meal)	(4)
Days since receipt × household size	-0.0002			
	(0.0013)			
Days since receipt of food stamps	-0.0034	-0.0037		
	(0.0048)	(0.0046)		
Household size	0.0183		-0.0087	0.0017
	(0.0228)		(0.0060)	(0.0081)
Demographics?	No	No	No	Yes
Sample	All	Single-person household	All	All
Model	OLS	OLS	Probit	Probit
Dataset	CSFII	CSFII	EBT	EBT
Number of observations	6652	519	1106	1103
(Pseudo-)R^2	0.0814	0.3523	0.0032	0.0442

Notes:

In regressions using the CSFII, each observation describes one household member on one survey day (out of three total days). In regressions using EBT data, each observation describes one respondent. Regressions (1) and (2) include dummies for survey day, day of week, year, month, and calendar date. In regressions (1) and (2), standard errors in parentheses are adjusted for intercorrelation within households. Demographic controls in column (4) include dummies for ten-year age categories, sex, race, and high school graduation status of household head, as well as total value of food stamps received in previous month. Dummies for missing data are included for all controls. Probit coefficients reported as marginal effects. Survey weights used as recommended by data providers.

Table 6: Inter-household Transfers Over the Month

	(1)	(2)	(3)	(4)
Dependent variable	Pr(Ate in other home)		Calories received	
Days since receipt of food stamps	0.0001 (0.0003)		0.4753 (0.4072)	
Short on cash		-0.0066 (0.0150)		-0.8644 (16.9869)
Model	Probit	Probit	OLS	OLS
No. of observations	6671	6683	6671	6683
(Pseudo-)R^2	0.1014	0.1017	0.0266	0.0273

Notes:

Data from CSFII. Each observation describes one household member on one survey day (out of three total days). All specifications include dummies for survey day, day of week, year, month, and calendar date. Standard errors in parentheses are adjusted for intercorrelation within households. Probit coefficients reported as marginal effects. Survey weights used as recommended by data providers.

8 Appendix

8.1 Summary Statistics

Appendix Table

Panel A: CSFII

	N	*Mean*	*Std. Dev.*	*Min.*	*Max.*
Energy (kcal)	6652	1638	813	24	10413
Age (years)	6652	24.8	21.9	0	91
Household size	6652	4.07	1.98	1	18
	N	*Percent*			
Male	6652	37.1			
White	6652	51.2			

Panel B: NFCS

	N	*Mean*	*Std. Dev.*	*Min.*	*Max.*
Food value ($)	309	61.4	38.9	.6	201.1
Household size	309	3.23	1.86	1	10

Panel C: EBT

	N	*Mean*	*Std. Dev*	*Min.*	*Max.*
Age	1095	38.6	1.5	18	93
Household size	1103	2.59	1.49	1	11
Amount of food stamps ($)	1103	183	111	0	760
	N	*Percent*			
Male	1103	17.0			
White	1103	33.3			
High school graduate	1103	46.9			

Notes:

Survey weights used as recommended by data providers. In CSFII data, characteristics refer to respondent. In Maryland EBT study, characteristics refer to household head.

8.2 Measurement Error and Functional Form

Consider a sample of individuals i with true caloric intake C_i^*. Let the true model of caloric intake be given by

$$c_i^* = \alpha - \beta D_i + \varepsilon_i$$

where $c_i^* \equiv \log C_i^*$, D_i is the number of days since receipt of food stamps, and ε_i is an i.i.d. error term orthogonal to D_i. Let C_i denote the reported caloric intake of individual i. We consider two cases.

Case 1 *Multiplicative measurement error.*

Suppose that reported intake C_i is related to true intake C_i^* by

$$C_i = \Phi_i C_i^*$$

with $\log \Phi_i$ orthogonal to D_i. Letting $\phi_i \equiv \log \Phi_i$ and $c_i \equiv \log C_i$, we can write

$$c_i = \alpha - \beta D_i + \varepsilon_i + \phi_i.$$

An OLS regression of c_i on D_i will therefore yield a consistent estimate for β. (Note, however, that if ϕ_i does not have mean 0 the resulting estimate of α will be biased.)

Case 2 *Additive measurement error.*

Now let reported intake be given by

$$C_i = C_i^* + \psi_i$$

where ψ_i is orthogonal to D_i. Then, by a first-order Taylor expansion around C_i^*, we can write

$$\begin{aligned} c_i &= \log\left(C_i^* + \psi_i\right) \\ &\approx \log C_i^* + \frac{\psi_i}{C_i^*} \\ &\approx \alpha - \beta D_i + \varepsilon_i + \frac{\psi_i}{C_i^*}. \end{aligned}$$

Since $\frac{\psi_i}{C_i^*}$ will not be orthogonal to D_i, an OLS regression of c_i on D_i will not yield a consistent estimate of β. In particular, since $\frac{\psi_i}{C_i^*}$ will tend to be larger when C_i^*

is smaller (and therefore when D_i is larger), OLS estimates will be biased upward relative to the true parameter β.

Using the level rather than the log of reported calories as the dependent variable gives

$$\begin{aligned} C_i &= C_i^* + \psi_i \\ &= \exp(\alpha - \beta D_i + \varepsilon_i) + \psi_i \\ &\approx \exp(\alpha) - \exp(\alpha)\beta D_i + \exp(\alpha)\varepsilon_i + \psi_i. \end{aligned}$$

Since ψ_i is orthogonal to D_i, an OLS regression of C_i on D_i will provide a consistent estimate of $\exp(\alpha)\beta$. Scaling the OLS coefficient by an estimate of the true mean of caloric intake will give a consistent estimate of the slope parameter β.

8.3 Market Value and Food Quality

Let the instantaneous utility from food consumption in period t be given by $u(F_t)$, where F_t is food consumption. Suppose further that F_t is produced from inputs $X_1, ..., X_n$ according to the production function $F(\bullet)$, which I will assume has constant returns to scale. The inputs have market prices $p_1, ..., p_n$, respectively.

The consumer's choice of foods X must satisfy the cost minimization program

$$\begin{aligned} &\min_{X_1,...,X_n} \sum_{i=1}^{n} p_i X_i \\ s.t.\ F_t &= F(X_1, ..., X_n) \end{aligned}$$

The first-order condition of this problem is that

$$p_i = \mu F_i$$

for all i, where μ is the shadow price of a marginal unit of food (which is a constant due to the assumption of CRS). Multiplying both sides by X_i and summing, we have that

$$\sum_{i=1}^{n} p_i X_i = \mu \sum_{i=1}^{n} F_i X_i.$$

By Euler's Theorem we can rewrite this expression as

$$\sum_{i=1}^{n} p_i X_i = \mu F_t.$$

Thus, the market value of food consumed is proportional to the composite good F_t that enters the instantaneous utility function $u(\bullet)$.

8.4 Uncertainty in the Exponential Model[21]

Adopting the model in section 5.1, let us add the possibility of some uncertainty in caloric intake. Then for any day t in the interior of the food stamp month intake C_t must satisfy

$$C_t^{-\rho} = \delta R E_t \left[C_{t+1}^{-\rho} \right].$$

Taking the well-known second-order approximation to this condition gives

$$E_t \left[\Delta c_{t+1} \right] = \frac{r - \gamma}{\rho} + \frac{\rho}{2} Var_t \left[\Delta c_{t+1} \right].$$

Solving for the discount rate γ gives

$$\gamma = r - \rho E_t \left[\Delta c_{t+1} \right] + \frac{\rho^2}{2} Var_t \left[\Delta c_{t+1} \right].$$

Since the second term in this expression must be weakly greater than zero, we can write

$$\gamma \geq r - \rho E_t \left[\Delta c_{t+1} \right].$$

So, we can estimate a lower bound of the daily discount rate necessary to explain the observed behavior of food stamp recipients as the product of the curvature parameter ρ and the average daily decline in food consumption, plus the interest rate r. Ignoring uncertainty will only lead to an underestimate of the true discount rate.

8.5 Equilibrium in the Quasi-hyperbolic Model

We wish to prove that consumption is proportional to wealth in the quasi-hyperbolic model set out in section 5.2.[22] The initial condition is obvious: $C_T = W_T$ by the budget constraint. Now suppose that $C_{t+1} = \alpha_{t+1} W_{t+1}$ for some time period $t+1$. By the generalized Euler equation (5) we know that

$$C_t^{-\rho} = \left[\alpha_{t+1} \beta \delta + (1 - \alpha_{t+1}) \delta \right] (\alpha_{t+1} W_{t+1})^{-\rho}.$$

Note that, from the budget condition, $W_{t+1} = W_t - C_t$. Substituting in this relationship gives

$$C_t^{-\rho} = \left[\alpha_{t+1} \beta \delta + (1 - \alpha_{t+1}) \delta \right] (\alpha_{t+1} (W_t - C_t))^{-\rho}.$$

Rearranging terms, one can show that

$$C_t = \frac{\alpha_{t+1}}{\alpha_{t+1} + \left(\delta \left(1 - (1 - \beta) \alpha_{t+1} \right) \right)^{\frac{1}{\rho}}} W_t.$$

[21] I am grateful to David Laibson for pointing out the argument in this section.

[22] See Laibson (1996) for a more complete derivation of this and other properties of the equilibrium consumption path under quasi-hyperbolic discounting and isoelastic utility.

By induction, then, we have that

$$C_t = \alpha_t W_t$$

for all t, with $\alpha_T = 1$ and

$$\alpha_t = \frac{\alpha_{t+1}}{\alpha_{t+1} + \left(\delta\left(1-(1-\beta)\,\alpha_{t+1}\right)\right)^{\frac{1}{\rho}}}$$

as desired.

8.6 Calculating the Welfare Costs of the Nutrition Cycle

For expositional purposes it will be helpful to consider welfare implications in continuous time, although the results are not sensitive to this modeling choice. Consider a household with a discount rate of 0 that faces a net interest rate of 0 and an instantaneous utility of consumption given by

$$u(t) = \frac{C(t)^{1-\rho}}{1-\rho}.$$

The household's optimal consumption path is is therefore constant on the interval $[0, T]$. Suppose, however, that given wealth W to spend over this interval the true consumption path is given by

$$C(t) = \exp(\theta - \upsilon t)$$

where υ is an exogenous slope parameter arising due to non-exponential discounting (or some other form of mistake). Then θ is determined by the budget condition

$$\int_0^T \exp(\theta - \upsilon t)\, dt = W$$

which implies that

$$\exp(\theta) = \frac{\upsilon W}{1-\exp(-\upsilon T)}.$$

We would like to know what share $(1-\lambda)$ of its wealth the household would be willing to sacrifice in order to achieve a constant consumption profile. In other words, we would like to find λ to solve the following equation:

$$\int_0^T \frac{(\exp(\theta - \upsilon t))^{1-\rho}}{1-\rho} dt = \int_0^T \frac{\left(\frac{\lambda W}{T}\right)^{1-\rho}}{1-\rho} dt.$$

We can simplify this expression to

$$\frac{1}{1-\rho}\left(\frac{\upsilon}{1-\exp\left(-\upsilon T\right)}\right)^{1-\rho}\frac{1}{\upsilon\left(1-\rho\right)}\left(1-\exp\left(-\upsilon\left(1-\rho\right)T\right)\right)=\frac{1}{1-\rho}\left(\frac{\lambda}{T}\right)^{1-\rho}T.$$

Note that the welfare loss is independent of wealth W. We can rewrite this expression as

$$\lambda=\left[\frac{1}{1-\rho}\left(\frac{\upsilon}{1-\exp\left(-\upsilon T\right)}\right)^{1-\rho}\frac{1}{\upsilon\left(1-\rho\right)}\left(1-\exp\left(-\upsilon\left(1-\rho\right)T\right)\right)\left(1-\rho\right)T^{-\rho}\right]^{\frac{1}{1-\rho}}.$$

Values for welfare losses reported in section 6 are obtained by evaluating this expression for $T=30$, $\upsilon=0.0040$, and alternative values for ρ.